Tribhuvan University

Institute of Science and Technology

2077

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BScCSIT Level/First Semester — Full Marks: 80
Mathematics[MTH 112] — Pass Marks: 32
Calculus — Time 3 Hrs.

Candidates are required to give their answers in their own words as far as practicable.

Group A ($10 \times 3 = 30$)

Attempt any **THREE** questions.

1. (a) If $f(x) = x^2$ then find $\frac{f(2+h)-f(2)}{h}$. [2]

   (b) (a) Dry air is moving upward. If the ground temperature is 20^o and the temperature at a height of $1km$ is 10^oC, express the temperature T in ^oC as a function of the height h (in kilometers), assuming that a linear model is appropriate. (b) Draw the graph of the function in part (a). What does the slope represent? (c) What is the temperature at a height of $2km$? [5]

   (c) Find the equation of the tangent to the parabola $y = x^2 + x + 1$ at $(0, 1)$. [3]

2. (a) A farmer has $2000ft$ of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area? [5]

   (b) Sketch the curve [3]

$$y = \frac{1}{x-3}$$

3. (a) Show that the $\int_1^\infty \frac{1}{x^2}$ converges and $\int_1^\infty \frac{1}{x}$ diverges. [2]

(b) If $f(x,y) = xy/(x^2+y^2)$, does f(x, y) exist, as $(x,y) \to (0,0)$? [3]

(c) A particle moves in a straight line and has acceleration given by $a(t) = 6t^2 + 1$. Its initial velocity is $4m/sec$ and its initial displacement is $s(0) = 5cm$. Find its position function $s(t)$. [5]

4. (a) Evaluate [5]

$$\int_{-3}^{2}\int_{0}^{\pi/2}(y + y^2\cos x)dxdy$$

(b) Find the Maclaurin's series for $\cos x$ and prove that it represents $\cos x$ for all x. [5]

Group B $(10 \times 5 = 50)$

Attempt any **TEN** questions.

5. If $f(x) = x^2 - 1, g(x) = 2x + 1$, find fog and gof and domain of fog.
6. Define continuity of a function at a point $x = a$. Show that the function $f(x) = \sqrt{1-x^2}$ is continuous on the interval $[-1, 1]$.
7. State Rolle's theorem and verify the Rolle's theorem for $f(x) = x^3 - x^2 - 6x + 2$ in $[0, 3]$.
8. Find the third approximation x_3 to the root of the equation $f(x) = x^3 - 2x - 7$, setting $x_1 = 2$.
9. Find the derivative of $\mathrm{r}(t) = (1+t^2)\mathrm{i} - te^{-t}\mathrm{j} + \sin 2t\mathrm{k}$ and find the unit tangent vector at $t = 0$.
10. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.
11. Solve: $y'' + y' = 0, y(0) = 5, y(\pi/4) = 3$
12. Show that the series $\sum_{n=0}^{\infty} \frac{1}{1+n^2}$ converges.
13. Find a vector perpendicular to the plane that passes through the points: $P(1,4,6), Q(-2,5,-1)$ and $R(1,-1,1)$
14. Find the partial derivative of $f(x,y) = x^3 + 2x^2y^3 - 3y^2 + x + y$, at $(2,1)$.
15. Find the local maximum and minimum values, saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.