Tribhuvan University

Institute of Science and Technology

2081

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Bachelor Level / First Year/ First Semester/ Science

Computer Science and Information Technology (MTH 117)

(Mathematics I)

(NEW COURSE)

Full Marks: 60
Pass Marks: 24
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Section A $(2 \times 10 = 20)$.

Attempt **any TWO** questions:

1. (a) Sketch the graph of $f(x) = x^2$. Find its domain and range. [1+2+2]
   (b) Evaluate: $\lim_{x \to 1} \sin^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$. [5]

2. (a) Where the function $f(x) = |x|$ is differentiable? Discuss. [5]
   (b) A farmer has 1200 m. of fencing and wants to fence off a rectangular field that boarders a straight river. He needs to fence along the river. What are the dimensions of the field that has the largest area? [5]

3. (a) Find the solution of the initial value problem $x^2\, y' + x\, y = 1,\ y(1) = 2,\ x > 0$. [5]
   (b) Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$. [5]

Section B $(8 \times 5 = 40)$.

Attempt **any EIGHT** questions:

4. Evaluate: $\int_0^{\sqrt{3}} \sqrt{1+x^2}\, x^3\, dx$. [5]

5. Find the Maclaurin series expansion of $f(x) = \sin x$ for all x. [5]

6. Find the unit normal and binormal vectors for the circular helix $\mathbf{r}(t) = \cos t\, \vec{i} + \sin t\, \vec{j} + t\, \vec{k}$. [4+1]

7. If $f(x,y) = \frac{x\,y}{x^2+y^2}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Justify. [5]

8. Determine whether the sequence $a_n = (-1)^n$ is convergent or divergent. [5]

9. The position vector of an object moving in a plane is given by $\mathbf{r}(t) = t^3\vec{i} + t^2\vec{j}$. Find its velocity, speed, and acceleration when $t = 1$ and illustrate geometrically. [2+1+1+1]

10. Show that every member of the family of function $y = \frac{1+c\,e^t}{1-c\,e^t}$ is a solution of the differential equation $y' = \frac{1}{2}(y^2 - 1)$. [5]

11. If $f(x,y) = 2x^3 - x^2y^3 - y^4$, find $f_x(1,-2), f_y(1,-1)$ and $f_{yx}(1,-1)$. [5]

12. Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ for 0 to 1. [5]