Tribhuvan University

Institute of Science and Technology 2080

X

Bachelor Level / First Year/ First Semester/ Science Computer Science and Information Technology (MTH112) (Mathematics I)

Full Marks: 80 Pass Marks: 32 Time: 3 hours.

(OLD COURSE)

Candidates are required to give their answers in their own words as for as practicable. The figures in the margin indicate full marks.

$$(3 \times 10 = 30).$$

Attempt any THREE questions:

1. (a) If
$$f(x) = 3x^2 - x + 2$$
, then find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$ and $f(0)$. [5]
(b) Find $\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$ [5]

2. (a) Find the derivative of
$$y = \frac{x^2 + x - 2}{x^3 + 6}$$
. [5] (b) Estimate the area between the parabola $y = x^2$ and the line $y = x$. [5]

(b) Estimate the area between the parabola
$$y = x^2$$
 and the line $y = x$. [5]

3. (a) Verify the Mean Value Theorem for the function
$$f(x) = 3x^2 + 2x + 5$$
 $x \in [-1,1].$

(b) Define initial value problem. Solve the equation
$$xy' + 2y = 3x$$
, $y(1) = 2$.

4. (a) Find the local maximum and minimum values and saddle points of
$$f(x,y) = x^4 + y^4 - 4xy + 1$$
. [5]

(b) Find the curvature of the parabola
$$y = x^2$$
 at the points $(0,0)$, $(0,1)$ and $(2,4)$. [5]

Group B
$$(10 \times 5 = 50)$$
.

Attempt any TEN questions:

5. Verify Rolle's theorem for
$$f(x) = x^2 - 4$$
, $x \in [-2, 2]$.

6. Find the Maclaurin series expansion of
$$\sin x$$
 at $x = 0$. [5]

7. If
$$f(x) = \sqrt{x-2}$$
 and $g(x) = \sqrt{x}$, find $(f \circ g)(x)$ and $(g \circ f)(x)$. [5]

8. Show that the absolute value function
$$f(x) = |x|$$
 is continuous everywhere. [5]

9. Find
$$\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-4x^2}} dx$$
. [5]

10. Sketch the curve
$$y = x^3$$
. [5]

11. Find the solution of
$$y'' + 4y' + 4 = 0$$
, $y(0) = 2$, $y'(0) = 1$. [5]

12. Test the series
$$\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$$
 diverges or converges. [5]

- 13. Define cross product of two vectors. If $\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k}$ and $\overrightarrow{b} = 2\overrightarrow{i} + 3\overrightarrow{j} 4\overrightarrow{k}$, find $\overrightarrow{a} \times \overrightarrow{b}$ and $\overrightarrow{b} \times \overrightarrow{a}$.
- 14. Find the second partial derivatives of $f(x,y) = x^3 + x^2y^3 2y^2$. [5]
- 15. Find the length of the arc of the semi-cubical parabola $y^2 = x^3$ between the points (1,1) and (4,8).