MTH.112-2075 ✡

Tribhuvan University
Institute of Science and Technology
2075
✡

Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (MTH. 112)
(Mathematics I)
(NEW COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt any three questions: (3×10=30)

1. (a) A function is defined by $f(x) = |x|$, calculate f(-3), f(4), and sketch the graph. (5)

   (b) Prove that the $\lim_{x\to 2} \frac{|x-2|}{x-2}$ does not exist. (5)

2. (a) Find the domain and sketch the graph of the function $f(x) = x^2 - 6x$. (5)

   (b) Estimate the area between the curve $y = x^2$ and the lines $y = 1$ and $y = 2$. (5)

3. (a) Find the Maclaurin series for cos x and prove that it represents cos x for all x. (4)

   (b) Define initial value problem. Solve that initial value problem of $y' + 2y = 3$, $y(0) = 1$. (4)

   (c) Find the volume of a sphere of radius a. (2)

4. (a) If $f(x,y) = \frac{y}{x'}$ does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Justify. (5)

   (b) Calculate $\iint_R f(x,y)dA$ for $f(x,y) = 100 - 6x^2y$ and R: $0 \le x \le 2$, $-1 \le y \le 1$. (5)

Attempt any ten questions: (10×5=50)

5. If $f(x) = \sqrt{2-x}$ and $g(x) = \sqrt{x}$, find fog and fof. (5)

6. Define continuity on an interval. Show that the function $f(x) = 1 - \sqrt{1-x^2}$ is continuous on the interval [-1, 1]. (5)

7. Verify Mean value theorem of $f(x) = x^3 - 3x + 2$ for [-1, 2]. (5)

8. Starting with $x_1 = 2$, fine the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$. (5)

9. Evaluate $\int_0^\infty x^3\sqrt{1-x^4}\,dx$. (5)

10. Find the volume of the resulting solid which in enclosed by the curve y = x and $y = x^2$ is rotated about the x – sxis. (5)

11. Find the solution of $y'' + 4y' + 4 = 0$ (5)

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12. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ converges or diverges. (5)

13. If **a** = (4, 0, 3) and **b** = (-2, 1, 5) find |**a**|, the vector **a** – **b** and 2**a** + 5**b**. (1+2+2)

14. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$. (5)

15. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$ (5)

$f(x,y) = x^2 + 2y^2$

$g(x,y) = x^2 + y^2 - 1 = 0$

$f_x = 2x$, $f_y = 4y$

$\vec{\nabla} f = \vec{i}\,2x + \vec{j}\cdot 4y$

$g_x = 2x$, $g_y = 2y$

$\vec{\nabla} g = \vec{i}\cdot 2x + \vec{j}\cdot 2y$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$2x\vec{i} + 4y\vec{j} = \lambda(2x\vec{i} + 2y\vec{j})$

$\Rightarrow 2x = 2x\lambda$ ——— (1)

$4y = 2y\lambda$ ——— (2)

$2x(1-\lambda) = 0$

$x = 0$ or $\lambda = 1$

$2y(2-\lambda) = 0$

$y = 0$ or $\lambda = 2$

$\lambda = 1$, $\lambda = 2$

C.P. (0, 0)