MTH. 155-2080 ✡

Tribhuvan University

Institute of Science and Technology

2080

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Bachelor Level / First Year /Second Semester/Science

Computer Science and Information Technology (MTH. 155)

(Linear Algebra)

(VERY OLD COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.
Attempt all questions:

Group A

(10×2=20)

1. What is system of linear equation? When the system is consistent and inconsistent?
2. Show that the vectors (1, 2) and (3, 4) are linearly independent.
3. Define invertible matrix with an example.
4. Prove that $(A+B)^T = A^T + B^T$, where A and B are matrices whose sizes arre appropriate for the above mentioned matrix.
5. Using Carmer's rule to solve the equations $3x + y = 5,\ \ 2x + 3y = 8$.
6. Define vector space with an example.
7. Let $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$ and let $u = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$, show that u belongs to the null space of A.
8. Determine whether the pair of vectors $u = \begin{bmatrix} 12 \\ 3 \\ -7 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -5 \\ 3 \end{bmatrix}$ are orthogonal or not.
9. Find the inverse matrix of $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$.
10. Consider a basis $\{b_1, b_2\}$ for $\mathbb{R}^2$ where $b_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, suppose an x in $\mathbb{R}^2$ has the coordinate vector $[x]_B$. Find x.

Group B

(5×4=20)

11. Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $u = (1, 0, -2)$ and $v = (1, 5, -2)$. If $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x) = Ax$, find $T(u)$ and $T(v)$.
12. Let $A = \begin{bmatrix} 5 & 4 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ -6 & 1 \end{bmatrix}$. Find AB and BA, if possible.
13. If v_1 and v_2 are the vectors of a vectors space V and W= Span $\{v_1, v_2\}$, then show that W is a subspace of V.
14. Find the eigenvalues of $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$.

15. Find an orthogonal projhection of x onto u where x=(7, 6) and u= (4,2).

OR

Show that v_1= (3, 1, 1), v_2= (-1,2, 1) and v_3 = (-1,-4, 7) are the orthogonal bases of $\mathbb{R}^3$.

Group C

(5×8=40)

16. Define basis and dimension of the vector space. Find the basis and dimension of the

subspace $H=\left\{\begin{bmatrix} a-36+6c \\ 5a+4d \\ b-2c-d \\ 5d \end{bmatrix} : a, b, c, d \in R\right\}$.

OR

Define T: $P_2 \to \mathbb{R}^2$ defined by $T(p) = \begin{bmatrix} p(0) \\ p(2) \end{bmatrix}$. Then

(a) Find the image of T of $p(t) = 3 + 5t + 7t^2$).
(b) Determine whether T is a linear transformation or not.

17. Let a = (1, -2, 5), b= (2, 5, 4) and c = (7, 4, -3) are the vectors. Determine whether c can be generated as a linear combination of a and b. That is, determine x_1 and x_2 exist such that $x_1a+x_2b=0$ has solution, find it.

18. Compute the multiplication of partitioned matrix for $A = \left[\begin{array}{ccc:cc} 1 & -3 & 2 & 1 & -51 \\ 2 & 3 & -2 & 3 & -1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 4 & 2 & 1 & 7 \end{array}\right]$ and

$B = \begin{bmatrix} 1 & 2 \\ 6 & 5 \\ 1 & 5 \\ \cdots & \cdots \\ 3 & 5 \\ 2 & 3 \end{bmatrix}$

19. Diagonalize the matrix $\begin{bmatrix} 7 & 4 & 6 \\ 2 & 5 & 0 \\ -2 & -2 & 3 \end{bmatrix}$, if possible.

20. Find the least-square solution of Ax= b for $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$.

OR

Let $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. Construct an orthogonal basis of $\mathbb{R}^3$.