Tribhuvan University

Institute of Science and Technology

2079

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Bachelor Level / First Year /Second Semester/Science

Computer Science and Information Technology (MTH. 163)

(Mathematics II)

(NEW COURSE)

Full Marks: 80

Pass Marks: 32

Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Group A

Attempt any THREE questions. (3×10=30)

1. Define system of linear equations. When a system of equations in consistent? Make echelon form to solve: (1+1+8)

$$-2x_1 - 3x_2 + 4x_3 = 5$$
$$x_2 - 2x_3 = 4$$
$$x_1 + 3x_2 - x_3 = 2$$

is consistent.

2. Define linear transformation with an example. (1+1+3+5)

$$\text{Let } A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix},\ \mathbf{v} = \begin{bmatrix} -2 \\ 1 \end{bmatrix},\ \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix},\ \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

and define a transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$ then

(a) find $T(\mathbf{v})$

(b) find $\mathbf{x} \in \mathbb{R}^2$ whose image under T is $\boldsymbol{b}$.

3. Find AB by block multiplication of the matrices (10)

$$A = \begin{bmatrix} 2 & -3 & 1 & 0 & -4 \\ 1 & -5 & -2 & 3 & -1 \\ 0 & -4 & -2 & 7 & -1 \end{bmatrix},\ B = \begin{bmatrix} 6 & 4 \\ 2 & -1 \\ -3 & 7 \\ 1 & 3 \\ 5 & -3 \end{bmatrix}$$

4. Find the least square solution of $A\mathbf{x} = \mathbf{c}$, where (10)

$$A = \begin{bmatrix} 1 & -3 & -3 \\ 1 & 5 & 1 \\ 1 & 7 & 2 \end{bmatrix},\ \mathbf{c} = \begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}.$$

and compute the associated least square error.

Group B

Attempt any TEN questions. (10×5=50)

5. Determine the column of the matrix A are linearly independent, where (5)

$$A = \begin{bmatrix} 3 & -3 & 6 \\ 0 & 2 & 4 \\ 0 & 3 & 0 \end{bmatrix}.$$

6. Let A= $\begin{bmatrix} 1 & 5 \\ -3 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value (s) of k, if any, will make AB=BA? (5)

7. Evaluate the determinant of the matrix (5)

$$\begin{bmatrix} 1 & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 3 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

8. When two column vectors in $\mathbb{R}^2$ are equal? Given an example. Compute $\mathbf{u} + 3\mathbf{v}, -\mathbf{u} - 2\mathbf{v}$, where

$$\mathbf{u} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \ \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}.$$ (1+4)

9. Prove that the two vectors **u** and **v** are perpendicular to each other if and only if the line through **u** is perpendicular bisector of the line segment from $-\mathbf{u}$ to $\mathbf{v}$. (5)

10. Find the eigenvalue of $A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$. (5)

11. Define null space of a matrix A. Let (1+4)

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -5 & 9 & -1 \end{bmatrix}, \text{ and } \mathrm{v} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix},$$

then show that v belongs to the null space matrix A.

12. Find the equation $y = a_0 + a_1 x$ of the least squares line that best fits the data points $(2, 1), \ (5, 2), \ (7, 3), \ (8, 3)$ (5)

13. Show that the solution of (5)

$$y_{k+2} - 4y_{k+1} + 3y_k = 0$$

are linearly independent.

14. Define group. Show that the set of integers is a group with respect to addition operation. (2+3)

15. Define ring and show that set of positive integers with respect to addition and multiplication operation is not a ring. (2+3)