CSC168-2080 ✡

Tribhuvan University

Institute of Science and Technology

2080

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Bachelor Level / First Year /Second Semester/Science

Computer Science and Information Technology (MTH 168)

(Mathematics II)

(NEW COURSE)

Full Marks: 60
Pass Marks: 24
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Group A $(2 \times 10 = 20)$.

Attempt **any TWO** questions:

1. What is a system of linear equations? When the system is consistent? Find a condition on g, h, k that makes the system consistent
$x_1 - 4x_2 + 7x_3 = g$
$3x_2 - 5x_3 = h$
$-2x_1 + 5x_2 - 9x_3 = k.$
[1+1+8]

2. Let $A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}, u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, b = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$. Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(x) = Ax$
   (a) Compute $T(u)$
   (b) Find a vector $x \in \mathbb{R}^3$ whose image under T is b.
   (c) Is x unique?
   [3+5+2]

3. Find a least square solution of $Ax = b$ where $A = \begin{bmatrix} 1 & -3 & -3 \\ 1 & 5 & 1 \\ 1 & 7 & 2 \end{bmatrix}, b = \begin{bmatrix} 5 \\ -3 \\ -5 \end{bmatrix}$. Also, compute the associated least square error.
[7+3]

Group B $(8 \times 5 = 40)$.

Attempt **any EIGHT** questions:

4. Are the vectors $v_1 = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 10 \\ 2 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} -5 \\ 0 \\ 6 \end{bmatrix}$ linearly independent? Justify.
[5]

5. Find LU factorization of $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 5 & 10 \\ 4 & 8 & 2 \end{bmatrix}$
[5]

6. Compute Det A where $A = \begin{bmatrix} 2 & -8 & 6 & 8 \\ 3 & -9 & 5 & 10 \\ -3 & 0 & 1 & -2 \\ 1 & -4 & 0 & 6 \end{bmatrix}$. [5]

7. Show that $H = \{(a - 3b, b - a, a, b) : a, b \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^4$. [5]

8. Is $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 5 & -3 \\ -4 & 9 \end{bmatrix}$? If so, find the eigenvalue. [3+2]

9. Let $u = (1, -2, 2, 0)$. Find a unit vector v in the same direction as u. [5]

10. Find the basis and dimension of Nul A where $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 7 & 8 \end{bmatrix}$. [3+2]

11. Define group. Show that $(\mathbb{Z}, .)$ doesn't form a group. [2+3]

12. Show that every field is an integral domain. [5]