MTH168-2081✡

Tribhuvan University

Institute of Science and Technology

2081

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Bachelor Level / First Year/ Second Semester/ Science

Computer Science and Information Technology (MTH 168)

(Mathematics II)

(NEW COURSE)

Full Marks: 60

Pass Marks: 24

Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Group A **(2 X 10 = 20)**

Attempt any **TWO** questions:

1. Define augmented matrix with an example. Find the general solution of the linear system whose augmented matrix is $\begin{bmatrix} 1 & 6 & 2 & -5 \\ -1 & 0 & 3 & 1 \\ 0 & -1 & -2 & 3 \end{bmatrix}$. [2+8]

2. (a) Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x) = AX$. Find the image under T of $u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $v = \begin{bmatrix} a \\ b \end{bmatrix}$. [5]

   (b) Prove that a map $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (y, x)$ is linear. [5]

3. Define inverse of a matrix. Find the inverse of a matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{bmatrix}$, if it exists. [1+9]

Group B (8 × 5 = 40).

Attempt any **EIGHT** questions:

4. Verify that $(AB)^{-1} = B^{-1}A^{-1}$ if $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$. [5]

[5]

5. Find LU factorization of $\begin{bmatrix} 2 & 5 \\ 6 & -7 \end{bmatrix}$. [5]

6. Compute the determinants by cofactor expansions $\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix}$. [5]

7. Show that the column vectors $u = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ and $x = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ is x in span $\{u, v, w\}$. [5]

8. Let $B = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ be bases for a vector space V, and suppose $b_1 = 6c_1 - 2c_2$ and $b_2 = 9c_1 - 4c_2$, then

   (a) find the change of coordinates matrix from B to C.

   (b) find $[x]_c$ for $x = -3b_1 + 2b_2$. [2.5+2.5]

9. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$. Find the eigenvalues and eigenvectors of A. [2+3]

10. Determine the least squares error in the least-square solution of $Ax = b$, where

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, \hat{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$ [5]

11. Prove that the binary operation $*$ defined on $\mathbb{Z}$ by letting $m * n = m + n + 1$ is commutative and associative. [2+3]

12. Show that $(\mathbb{Q}, +, .)$ forms a ring. [5]