MTH163-2081✡

Tribhuvan University

Institute of Science and Technology

2081

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Bachelor Level / First Year/ Second Semester/ Science
Computer Science and Information Technology (MTH 163)
(Mathematics II)
(OLD COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Group A $(3 \times 10 = 30)$.

Attempt any **THREE** questions:

1. What is a system of linear equations? When the system is consistent? Determine x_1, x_2, x_3 if the system
$x_1 - 2x_2 + x_3 = 3$
$2x_1 + 3x_2 - x_3 = 5$
$3x_1 + x_2 + 2x_3 = 4$ is consistent. [1+1+8]

2. (a) Find the standard matrix A for the dilation transformation $T(x) = 2x$ for $x \in \mathbb{R}^2$. [5]
(b) Prove that a map $T : \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = x - y$ is linear. [5]

3. Find a least square solution of $Ax = b$ where $A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 2 & 3 \end{bmatrix}, b = \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix}$. [10]

4. What do you mean by LU factorization? Find LU factorization of $\begin{bmatrix} 1 & 2 & 5 \\ 6 & 3 & 8 \\ -5 & 2 & 3 \end{bmatrix}$. [2+8]

Group B $(10 \times 5 = 50)$.

Attempt any **TEN** questions:

5. Define inverse of a matrix. Find the inverse of $A = \begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix}$. [1+4]

6. When two column vectors in $\mathbb{R}^2$ are equal? Give an example. Compute $-u + 2v$ and $3v - 4u$, where $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$. [1+1+1+2]

7. Evaluate: $\begin{vmatrix} 2 & 5 & 4 & 1 \\ 4 & 7 & 6 & 2 \\ 6 & -2 & -4 & 0 \\ -6 & 7 & 7 & 0 \end{vmatrix}$. [5]

8. Find the eigenvalue and eigenvectors of $\begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$. [2+3]

9. Let $u = (2, -2, 0, 1)$. Find a unit vector v in the same direction as u. [5]

10. Find the basis for Null A and column A for the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 1 & 4 & -3 & -2 \\ 1 & -1 & 2 & 3 \end{bmatrix}$. [5]

11. Compute $\frac{u.v}{u.u}$ and $\left(\frac{u.v}{v.v}\right) u$ if $u = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$. [2+3]

12. Show that $\{t, 1-t, 1+t-t^2\}$ is a basis for $\mathbb{P}_2$. [5]

13. Verify that $1^k, (-2)^k, 3^k$ are linearly independent signals. [5]

14. Define group. Prove that $(\mathbb{Z}, +)$ forms a group. [2+3]

15. Compute the product (11)(-4)of the ring $\mathbb{Z}_{15}$. [5]