CSc.163-2076 ✡

Tribhuvan University

Institute of Science and Technology

2076

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Bachelor Level / First Year /Second Semester/Science

Computer Science and Information Technology (MTH. 163)

(Mathematics II)

<u>(NEW COURSE)</u>

Full Marks: 80

Pass Marks: 32

Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.

The figures in the margin indicate full marks.

Group- A $\qquad$ $3\times10=30$

Attempt any **THREE** questions.

1. When a system of linear equation is consistent and inconsistent? Give an example for each. Test the consistency and solve the system of equations: $x-2y=5$, $-x+y+5z=2$, $y+z=0$. [2+2+6]

2. What is the condition of a matrix to have an inverse? Find the inverse of the matrix $A=\begin{bmatrix}5 & 1 & 2\\ 1 & 0 & 3\\ 4 & -3 & 8\end{bmatrix}$, if it exists. [2+8]

3. Find the least- square solution of $Ax=b$ for $A=\begin{bmatrix}1 & -6\\ 1 & -2\\ 1 & 1\\ 1 & 7\end{bmatrix}$ and $b=\begin{bmatrix}-1\\ 2\\ 1\\ 6\end{bmatrix}$. [10]

4. Let T is a linear transformation. Find the standard matrix of T such that

   (i) $T: \mathbb{R}^2 \to \mathbb{R}^4$ by $T(e_1)=(3, 1, 3, 1)$ and $T(e_2)=(-5, 2, 0, 0)$ where $e_1=(1, 0)$ and $e_2)=(0, 1)$;

   (ii) $T: \mathbb{R}^2 \to \mathbb{R}^4$ rotates point as the origin through $\frac{3\pi}{2}$ radians counter clockwise.

   (iii) $T: \mathbb{R}^2 \to \mathbb{R}^4$ is a vertical shear transformation that maps e_1 into e_1-2e_2 but leaves vector e_2 unchanged. [4+3+3]

$X^T X A = X^T B$

Group- B $\qquad$ $10\times5=50$

Attempt any **TEN** questions.

5. For what value of h will y be in span $\{v_1, v_2, v_3\}$ if $v_1=\begin{bmatrix}1\\ -1\\ -2\end{bmatrix}$, $v_2=\begin{bmatrix}5\\ -4\\ -7\end{bmatrix}$, $v_3=\begin{bmatrix}-3\\ 1\\ 0\end{bmatrix}$ and $y=\begin{bmatrix}-4\\ 3\\ h\end{bmatrix}$? [5]

6. Let us define a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x)=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}-x_2\\ x_1\end{bmatrix}$. Find the image under T of $u=\begin{bmatrix}4\\ 1\end{bmatrix}$, $v=\begin{bmatrix}2\\ 3\end{bmatrix}$, and $u+v=\begin{bmatrix}6\\ 4\end{bmatrix}$. [5]

7. Let A $=\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. Determine the value (s) of k if any will make AB= BA. [5]

8. Define determinant. Compute the determinant without expanding $\begin{vmatrix} -2 & 8 & -9 \\ -1 & 7 & 0 \\ 1 & -4 & 2 \end{vmatrix}$. [1+4]

9. Define null space. Find the basis for the null space of the matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$. [1+4]

10. Let B= $\{b_1, b_2\}$ and C= $\{c_1, c_2\}$ be bases for a vector space V, and suppose $b_1= -c_1+ 4c_2$ and $b_2= 5c_1-3c_2$. Find the change of coordinate matrix for a vector space and find $[x]_C$ for x= $5b_1+3b_2$. [2.5+2.5]

11. Find the eigenvalues of the matrix $\begin{bmatrix} 6 & 5 \\ -8 & -6 \end{bmatrix}$. [5]

12. Find the QR factorization of the matrix $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$. $A = QR$ $R = Q^T A$ [5]

13. Define binary operation. Determine whether the binary operation * is associative or commutative or both where * is defined on $\mathbb{Q}$ by letting $x*y=\frac{x+y}{3}$. [5]

14. Show that the ring $(\mathbb{Z}_4, +_4, \cdot_4)$ is an integral domain. [5]

15. Find the vector x determined by the coordinate vector $[x]_\beta = \begin{bmatrix} -4 \\ 8 \\ -7 \end{bmatrix}$ where

$\beta = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}$. [5]